Paper Reference(s)

9801

Edexcel GCE

Mathematics

Advanced Extension Award

Thursday 27 June 2002 – Afternoon

Time: 3 hours

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

Full marks may be obtained for answers to ALL questions.
In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title, the paper reference (9801), your surname, other names and signature.
Answers should be given in as simple a form as possible, e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae including Statistical Formulae and Tables' is provided.
This paper has seven questions. Pages 6, 7 and 8 are blank.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. Solve the following equation, for $0 \leq x \leq \pi$, giving your answers in terms of π.

$$\sin 5x - \cos 5x = \cos x - \sin x.$$

(8)

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2. In the binomial expansion of

$$(1 - 4x)^p, \quad |x| < \tfrac{1}{4},$$

the coefficient of x^2 is equal to the coefficient of x^4 and the coefficient of x^3 is positive.

Find the value of p.

(9)

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3. The curve C has parametric equations

$$x = 15t - t^3, \;\; y = 3 - 2t^2.$$

Find the values of t at the points where the normal to C at (14, 1) cuts C again.

(11)

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4. Find the coordinates of the stationary points of the curve with equation

$$x^3 + y^3 - 3xy = 48$$

and determine their nature.

(14)

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5. **Figure 1**

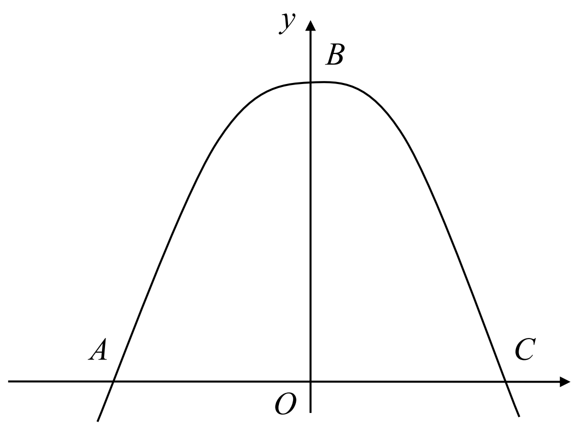

Figure 1 shows a sketch of part of the curve with equation

$$y = \sin(\cos x).$$

The curve cuts the x-axis at the points A and C and the y-axis at the point B.

(a) Find the coordinates of the points A, B and C.

(3)

(b) Prove that B is a stationary point.

(2)

Given that the region OCB is convex,

(c) show that, for $0 \le x \le \frac{\pi}{2}$,

$$\sin(\cos x) \le \cos x$$

and

$$\left(1 - \frac{2}{\pi}x\right) \sin 1 \le \sin(\cos x)$$

and state in each case the value or values of x for which equality is achieved.

(6)

(d) Hence show that

$$\frac{\pi}{4} \sin 1 < \int_0^{\frac{\pi}{2}} \sin(\cos x) \, dx < 1.$$

(4)

6. **Figure 2**

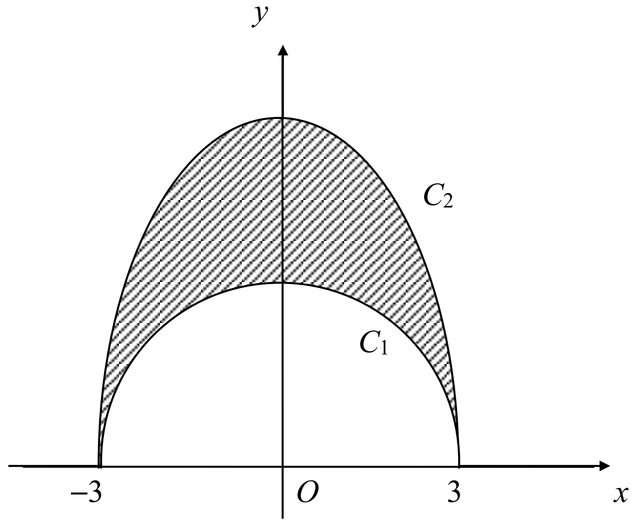

Figure 2 shows a sketch of part of two curves C_1 and C_2 for $y \geq 0$.

The equation of C_1 is $y = m_1 - x^{n_1}$ and the equation of C_2 is $y = m_2 - x^{n_2}$, where m_1, m_2, n_1 and n_2 are positive integers with $m_2 > m_1$.

Both C_1 and C_2 are symmetric about the line $x = 0$ and they both pass through the points (3, 0) and (–3, 0).

Given that $n_1 + n_2 = 12$, find

(*a*) the possible values of n_1 and n_2 ,

(4)

(*b*) the exact value of the smallest possible area between C_1 and C_2, simplifying your answer,

(8)

(*c*) the largest value of x for which the gradients of the two curves can be the same. Leave your answer in surd form.

(5)

7. A student was attempting to prove that $x = \frac{1}{2}$ is the only real root of

$$x^3 + \tfrac{3}{4}x - \tfrac{1}{2} = 0.$$

The attempted solution was as follows.

$$x^3 + \tfrac{3}{4}x = \tfrac{1}{2}$$

$\therefore$ $$x(x^2 + \tfrac{3}{4}) = \tfrac{1}{2}$$

$\therefore$ $$x = \tfrac{1}{2}$$

or $$x^2 + \tfrac{3}{4} = \tfrac{1}{2}$$

i.e. $$x^2 = -\tfrac{1}{4} \qquad \text{no solution}$$

$\therefore$ only real root is $x = \frac{1}{2}$

(*a*) Explain clearly the error in the above attempt.

(2)

(*b*) Give a correct proof that $x = \frac{1}{2}$ is the only real root of $x^3 + \frac{3}{4}x - \frac{1}{2} = 0$.

(3)

The equation

$$x^3 + \beta x - \alpha = 0 \qquad \text{(I)}$$

where α, β are real, $\alpha \neq 0$, has a real root at $x = \alpha$.

(*c*) Find and simplify an expression for β in terms of α and prove that α is the only real root provided $|\alpha| < 2$.

(6)

An examiner chooses a positive number α so that α is the only real root of equation (I) but the incorrect method used by the student produces 3 distinct real "roots".

(*d*) Find the range of possible values for α.

(7)

Marks for style, clarity and presentation: 7
TOTAL FOR PAPER: 100 MARKS

END